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Mapping mean and variance of runoff in a river basin

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Abstract

The study presents an approach to depict the two first order moments of runoff as a function of area (and thus on a map). The focal point is the mapping of the statistical properties of runoff $q=q(A, D)$ in space (area A) and time (time interval D). The problem is divided into two steps. Firstly the first order moment (the long term mean value) is analysed and mapped applying an interpolation procedure for river runoff. In a second step a simple random model for the river runoff process is proposed for the instantaneous point runoff normalised with respect to the long term mean. From this model theoretical expressions for the time-space variance-covariance of the inflow to the river network are developed, which then is used to predict how the second order moment vary along rivers from headwaters to the mouth. The observation data are handled in the frame of a hydrological information system HydroDem, which allows displaying the results either in the form of area dependence of moments along the river branches to the basin outlet or as a map of the variation of the moments across the basin space. The findings are demonstrated on the example of the Moselle drainage basin (French part).

1 Introduction

Mapping statistical parameters of runoff across space constitutes one of the fundamental tasks in hydrology. As the probabilistic characteristics of the runoff formation process are a priori unknown, the only direct source of information for solving this task is hydrological observations series. Hydrological series, however, may be too few and/or too short for a reliable determination of quantitative runoff characteristics, or even not available at all. In this case hydrology turns to indirect approaches for the study of runoff distribution in space, namely the “geographical interpolation” (in some sense) of the parameters of the runoff distribution. This is done utilizing hydrological series for different sites in a large river basin or different river basins within a region.

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Such studies fall under what is vaguely defined as “regional hydrology”, to which this paper is a contribution.

5 The main hydrometeorological processes (rainfall, evapotranspiration, temperature etc.) when observed at the land surface develop in a three-dimensional space – the two geographical coordinates (x, y) and time t . The variation of these variables across space is described as contour (isoline) maps in classical works, i.e. they are space-filling phenomena and allow straightforward interpolation. River discharge (surface runoff), on the other hand, is formed in a two-dimensional space – the distance along a river l_A (related to the point with area A in a basin) and time t . This relationship to
10 the area (and not the (x, y) coordinates) explains why the variation of runoff characteristics across space, determined from discharge measurements, cannot be exposed as a contour map as simply as hydrometeorological variables. Neither can it be directly claimed that it is a space-filling phenomenon.

The contradiction in mapping runoff characteristics was noted early by Gergov (1972) with respect to the runoff “module”. This latter concept, very often found in Central and East European hydrological literature, can have three interpretations – 1) the true specific runoff q_1 with which a point (x, y) in a basin contributes to the runoff in the river (rainfall excess; in the general case this might not be a space filling variable); 2) the areal mean runoff in a point l_A in a river derived as $q_2=Q/A$ (where Q is the
20 mean annual discharge and A the area of the corresponding basin); 3) the derivative of the specific runoff $q_3=dQ/dA$, i.e. the contribution of runoff to the river reach for each increment in the area. For all cases the dimension of runoff is volume per area and time $[L^3/(L^2 T)]$, and in hydrology either $[l/km^2/s]$ or $[mm/year]$ is used. Herein the latter of the two will be used.

25 This study presents an approach to depict runoff as a function of area (and thus on a map) though not in terms of a deterministic relationship but on how its statistical properties (mean value and variance) develop with area. The approach does not fully solve the theoretical ambiguities in mapping runoff but permits a reasonable simplification of the problem. We thus accept that runoff can be looked upon as a random variable in

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the two dimensional space $q=Q/A=q(I_A, t)=q(A, t)$. Consequently we accept the second definition given earlier, which also implies that runoff is only defined along a river network. We further note that runoff characteristics can be expressed for an instance t in time only in theory. In reality observations of discharge allow us to estimate runoff for a certain time resolution D in time (an hour, a day, five days, a month, a year). Mapping runoff characteristics involves thus mapping of statistical properties (moments) of $q=q(A, D)$ in space A and time D which is the focus for the present study. This task is similar to studying scale dependence in moments of runoff in space A and time D .

The mean value as well as the variance show large variability across space revealing a non-homogeneous random process. In general there exists a great coherence between the variability of these two moments i.e. a significant part of the variability of the variance might be explained by the pattern of variability of the mean. The mean value is independent of the used time interval D , while this is not the case for the variance. The variance is reduced in relative terms the more the longer the time interval D is. Furthermore, the variance reduces also in relative terms due to averaging over the basin area (the support of the random process), the larger the basin area the more reduction. The full temporal-spatial covariance structure of the runoff process needs to be known to be able to evaluate this variance reduction due to time and space averaging.

The patterns of spatial variability of the mean and variance of runoff that can be identified from observations are of course influenced by the fact that these observations represent averaged values. To be able to map these quantities we need to assume the existence of an instantaneous point runoff (IPR) process. The task is thus to solve the inverse problem of identifying this process from the observations representing averages in time and space. In a second step we are able to average this IPR process along rivers to achieve the desired map.

For the mean values this task is rather straightforward and basically is a problem of stochastic interpolation with local support or in another vocabulary block kriging. An obstacle might be the complexity in the structure of the covariance of runoff as the data represent a mixture of nested and non-nested basins. This problem has

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been studied by Gottschalk (1993a) and the theoretical findings of in this study will be brought forward here. Gottschalk (1993b) introduced a method for stochastic interpolation of runoff along the river network with a constraint preserving the water balance, i.e. at each downstream point in the river the runoff is the sum of the upstream inflow.

Sauquet (2000) and Sauquet et al. (2000) developed this methodology further and combined it with a system for structuring hydrographical networks in a hierarchical way called HydroDem (Leblois and Sauquet, 2000). It allows an effective reconstruction of the variation of mean annual runoff (first order moment) along the river network in a basin from discharge observations and a DEM. This latter interpolation scheme will be followed here to map the mean value. The resolution of the underlying DEM defines the size of computational units (grid cells, sub-basins). It is further assumed that each unit contains a segment of a river (“a flow path”). The difference between the definition of runoff “module” (specific runoff) q_1 and q_2 is eliminated by this assumption and all the territory is assigned a runoff value down to the scale of basic computational units.

The pattern of variability of the variance identified from observations is still more complex than in case of the mean. There are two sources of variability. The first one is a reflection of the natural variability of the IPR process. This is then overlain by variability induced by the variance reduction which varies with the support i.e. the basin area. This part of the variability might constitute a significant part of the total variability. As such it introduces a dependence on the basin area. In principle, it would be possible to also solve this problem with stochastic interpolation with local support. A constraint can be added so that the sum of variance-covariances over sub-basins should add up to the total variance over the whole basin. The authors of this paper have made several attempts in this direction but they have all failed. The reason is mainly due to inconsistencies in observed data which does not fulfil the proposed constraint.

The alternative developed herein is as follows. Firstly we benefit from the fact that the variation pattern of the mean value is also reflected in the variance by introducing a new variable namely the normalized instantaneous point runoff i.e. IPR divided by the point mean. For this latter variable a parametric random model is proposed that

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allows constructing a space-time covariance structure for the runoff along rivers and specifically map the variance along rivers.

The paper is structured as follows. In the first section of the paper the statistical characteristics of runoff data from the Moselle basin (France) are described which is used to test the approach. In the next section the method for interpolation of the mean is described and the random model for river runoff in a basin is proposed, which allows calculating variance functions, reductions factors, auto- and cross-correlation. The derivation of these latter functions is rather laborious and is only briefly presented in Appendix B. The derived theoretical expressions are then used to model the spatial and temporal scale dependence in first and second order moments, the autocorrelation at a site, and the cross-correlation between sites along rivers of the Moselle basin. Examples of derived maps of mean annual runoff and coefficient of variation of runoff of different duration are shown in Appendix A. The paper ends with a discussion of the results and conclusions.

2 Runoff characteristics of the Moselle river basin

The study area covers the French part of the Moselle basin, one of the main tributaries to the Rhine River. The Moselle River basin demonstrates a great variability of the landscape due to the heterogeneity of the geology (crystalline and sedimentary rocks, schist, sandstone) and relief under continental climate. The headwaters are located in the Vosges Mountains covered by forests whereas downstream parts are lowlands (alluvial plains) with a landscape influenced by agricultural practices.

A map of the Moselle basin and the drainage pattern identified by HydroDem as well as the discharge stations are shown in Appendix A. It is complemented by a schematised river network with its eight main branches (Fig. 1). The size of the main basin is 9387 km² at its outlet at Hauconcourt, France. Within this basin another 16 sub-basin with areas in the range 73–3350 km² have been utilised. The discharge data used are the breakpoints and it thus allows the estimation of runoff averaged over any duration

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D. Here five durations have been considered, namely: an hour, a day, five days, a month and a year.

The study focuses on the two first order moments. The first order moment around zero of the Moselle discharge data are plotted against basin area in a double logarithmic diagram in accordance with the traditional way of representing scale dependences in Fig. 2. There is a tendency that the data cluster around straight lines but the scatter is quite large. The first order moments (the long term mean values) do not depend on the duration *D*, which is the case for higher order moments. This spatial-temporal dependence for the second order moment is illustrated in Fig. 3 in terms of the coefficient of variation. For short durations (an hour and a day) this coefficient shows a strong decay with the area. This dependence with area decreases with the increase in duration and for annual values it is negligible. For small catchments the estimated values differ between one hour and one day data. This difference is almost eliminated for basins bigger than 1000 km².

Figure 4 offers an illustration of the empirical auto- and cross-correlations at and between runoff stations along the tributary to the Moselle viz. the Meurthe River (branch 4, cf. Fig. 1) and the outlet site on the Moselle River at Hauconcourt (station a793061). It shows the estimated autocorrelation function for the central site in this river branch (station a627101) for the four different durations considered (a), the same function for one hour duration for the five sites along the same river branch (b), the cross-correlation between the central and outlet site for different durations (c) and the cross-correlation between the site at the outlet and those upstream (d).

There is a strong autocorrelation in data for small time intervals (hours, days), which then decreases more rapidly towards larger time scales. However, we can note that the decay of the correlation is not of the exponential type frequently used in hydrology. A heavy tail is observed for large time scales. We interpret this as the existence of (at least) two characteristic scales – one of the order of one day and another of the order of one month. Examining the autocorrelation at a site (Fig. 4a) we note that the correlation increases with increasing time scale from hours, to days, five days and

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months. One would expect a similar although less pronounced behaviour when moving downstream and the basin size is increased (Fig. 4b). For very small time steps (hours) this is actually the case. At a larger scale the tail of the correlation function does not show any clear systematic pattern corresponding to the basin scale, although there exists a slight tendency of an increased memory when moving downstream along the branch. The impression is that the weight of the heavy tailed part of the correlation varies locally.

The empirical cross-correlation functions show a drop (“nugget”) in the correlation at lag zero, which is larger the larger the difference in basin size is. At a site this drop is decreased when the duration increases (Fig. 4c). However, the behaviour at short time lags can be quite complex – sometimes with a small increase before decaying and in other cases an immediate decay (Fig. 4d). For the time being it has not been possible to explain all the details in the behaviour of the correlation functions.

The spatial correlation, revealed by Fig. 4d for individual sites, would in principle allow showing the plot of spatial correlation coefficients along and between river branches against some distance measure. The structure of such a diagram is, however, quite complex. Runoff as related to points along rivers is a non-homogeneous process (Gottschalk, 1993a). Furthermore, nested and non-nested sub-basins show drastic differences in correlation, very high in the first case and lower in the other. Finally, the distance measure is not obvious in this case. Should it be an Euclidian distance between observation sites, distance between centres of gravity of basins, distance along rivers between sites etc.? Whatever the used measure, the result is a scatter of points without structure. For this reason this type of plots is avoided here.

3 Long-term mean annual runoff

Let $X(l, t)$, the inflow to a river at a length coordinate l and in time t , represent a two dimensional random field. We allow the long term mean value $m_X(l)$ to vary in space l but let it be constant in time t . We thus admit that that the runoff production

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systematically can vary from one locality to another in a basin, i.e. the generic process is non-homogeneous in space. On the other hand stationarity is postulated in time. In reality runoff shows seasonal variations with more or less stable patterns (Krasovskaia and Gottschalk, 1992). We will anyhow accept this as a simplification for the time being.

The long term mean value $m_X(l)$ characterises the variation in the intensity in runoff formation across space being a product of the local landscape and climatic features. This first order moment is not influenced by the dynamics of the runoff formation process as the higher order moments are. We are thus able to map $m_X(l)$ with a due consideration of the fact that it is a variable in a one dimensional space l , the coordinate along the river network applying stochastic interpolation with local support. We here will follow the approach developed by Sauquet et al. (2000). Firstly a theoretical model of an assumed point process is estimated from the empirical covariance function. This point covariance function is then used for interpolation of runoff to each grid cell of the Moselle basin following a hierarchical scheme to be able to fulfil a water balance constraint along the river.

The covariance between two sites in a nested river system was earlier treated by Gottschalk (1993a). A simple exponential function is postulated for the correlation of the inflow to the river between to points l_1 and l_2 along the river at a distance $\lambda = |l_1 - l_2|$: $\rho(\lambda) = \exp(-\lambda/K)$. The constant K has the dimension of length and describes the characteristic scale of variation of runoff formation and reflects landscape and long-term climatic features. An approximate expression for a corresponding covariance between two nested basins is derived as:

$$\text{Cov}(A_1, A_2) = \text{Cov}(L_{A_1}, L_{A_2}) = 2\sigma_X^2 \left(\frac{K^2}{L_{A_1} L_{A_2}} \right) \left\{ \frac{L_{A_1}}{K} + \frac{1}{2} e^{-L_{A_1}/K} + \frac{1}{2} \left(1 - \frac{L_{A_1}}{K} + \frac{L_{A_2}}{K} \right) \left(e^{-L_{A_2}/K} - e^{-(L_{A_2}-L_{A_1})/K} \right) - \frac{1}{2} \right\} \quad (1)$$

A_1 and A_2 denote the two areas of sub-basins, where the first is nested within the second, drained by the river segments of lengths L_{A_1} and L_{A_2} ($L_{A_1} < L_{A_2}$), where the shorter one is common. Let us specifically look at the situation when $L_{A_1} = L_{A_2} = L_A$, i.e. we get

an expression for the variance over the sub-basins. Insertion into the equation above yields:

$$Var(A) = \sigma_A^2 = 2\sigma_X^2 \left(\frac{K}{L_A} \right)^2 \left\{ \frac{L_A}{K} - 1 + e^{-L_A/K} \right\} \quad (2)$$

Whether we use area or distance the expression for the covariance between two sub-basins along the same river anyhow indicates a non-homogeneous process, i.e. it depends on the absolute coordinates $A_1(L_{A1})$ and $A_2(L_{A2})$ and not the relative difference between the positions of the points with these coordinates. The covariance along a river is thus represented by an ensemble of curves conditioned on the position of the downstream site $A_2(L_{A2})$ as a function of the difference in area (position). For covariance between sites in non-nested basins it is not possible to develop close form analytical expressions, but it must be derived numerically as a conventional covariance with local support.

The details of parameter estimation and interpolation follow Sauquet et al. (2000). The characteristic space scale was estimated to $K=50$ km (cf. Eq. 1) and the spatial point standard deviation to $\sigma_X=300$ mm/year. The resulting dependence with basin area is shown in Fig. 5 and the corresponding map in Appendix A. The figure shows the empirical data points arranged along river branches. The HydroDem software (Leblois and Sauquet, 2000), utilised in this study, allows a retrieval of a unique string that arranges the grid cells on a map in accordance with the structure of the river network, thus linking the position of the grid cell with the integrated area of the basin at this position. Accordingly, we are able to plot the value at each grid cell of the map against area and they are shown as a grey background in the figure. When scale dependence is studied in the context of a river basin the scale relations develop along river branches towards a common value at the outlet (this is guaranteed due to the water balance constraint in the interpolation scheme). The diagram in Fig. 5 reveals the relative contribution from the branches to the total discharge. The flow in the main river can therefore be higher (lower) than the surrounding contribution due to high (low) inflow

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upstream.

4 Normalised instantaneous point runoff

In a second step we now turn to the second order moments with the assumption that the variation in $m_X(l)$ is known. To achieve homogeneity in space the original process is normalised with respect to this long term mean:

$$Z(l, t) = X(l, t) / m_X(l) \quad (3)$$

This homogeneous random field in time and space $Z(l, t)$ is the main focus for the study with the aim of developing a random model for this variable and specifically deriving expressions for its variance-covariance function.

For the general formulation of a time-space process Vanmarcke (1988) distinguishes between three important special types of two dimensional covariance functions, namely:

- the covariance structure is separable
- the correlation structure is isotropic, i.e. the covariance structure can be expressed in terms of the “radial” covariance function
- the covariance structure is ellipsoidal, i.e. by appropriate scaling and rotation of the coordinate axes random fields with ellipsoidal covariance structure can be reduced to isotropic random fields.

Gandin and Kagan (1976) suggest a covariance model similar to the second type for use in meteorology and climatology:

$$\text{Cov}[\lambda, \tau] = \sigma^2 \rho(|(\lambda/\nu) + \tau|) \quad (4)$$

where λ as before is the relative distance between to points in the river, τ the time lag and ν is a velocity and h/ν can be interpreted as a time of travel. The expression has

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its root in the so called Taylor's hypothesis for turbulent flow also known as the Taylor "frozen turbulence" hypothesis. We will here assume that the process $Z(l, t)$ has this type of isotropic space-time covariance function.

$Z(l, t)$ describes the instantaneous inflow at a point in a river. The important variable is the integrated process $\bar{Z}(A, t)$, i.e. the normalised discharge of a river with a basin of size A . We derive it by integration of $Z(l, t)$ from the most distant point in the river ($l=0$) down to its outlet at l_A . In accordance with the formulation of the covariance function Eq. (4) we replace the distance by the time of travel (time of concentration) $T_{CA}=L_A/v$ along the river distance L_A . The integrated process is thus expressed by:

$$\bar{Z}(A, t) = \bar{Z}(T_{CA}, t) = \frac{1}{T_{CA}} \int_{t-T_{CA}}^t Z(t', t) dt' = \frac{1}{T_{CA}} \int_t^{t+T_{CA}} Z(t-t') dt' \quad (5)$$

This formula coincides in form with a system interpretation of the rational method (Singh, 1988, p. 123) if runoff is expressed in terms of flow per area, i.e. a unit pulse of duration T_{CA} and height $1/T_{CA}$. There is of course an important difference in the fact that we here deal with the inflow to the river and not directly the rainfall like in the rational method. It is anyhow a simplification that neglects the dynamics and non-linearity of river flow. The only argument for doing this is the Principle of Parsimony as formulated by Tukey (1961): it may pay not to try to describe in the analysis the complexities that are really present in the situation. He stresses the importance of re-considering a model structure towards a simpler representation, which might improve the performance of the estimation method.

The mean value of the process $\bar{Z}(T_{CA}, t)$ will be independent of time of concentration as it represents a normalised value (cf. Eq. 3). The variance-covariance, however, will change with changing time of concentration. The covariance function for the integrated

process is derived as:

$$\text{Cov}(\bar{Z}(T_{CA}, t), \bar{Z}(T_{CA}, t + \tau)) = \text{Cov}_A(\tau) = \frac{1}{T_{CA}^2} \int_t^{t+T_{CA}} \int_{t+\tau}^{t+T_{CA}+\tau} \sigma_Z^2 \rho(t' - t'') dt' dt'' \quad (6)$$

This type of double integral can be easily transformed to a single integral by a simple variable transformation (e.g. Gottschalk, 1993a). A more general method for simplification is to apply the following relation between the covariance for areas (lines as special cases) and the underlying point covariance (Matérn, 1960):

$$\text{Cov}_A(\tau) = \int_{\min(h)}^{\max(h)} \text{Cov}(|h|) f(h) dh = E[\text{Cov}(|h|)] \quad (7)$$

where $f(h)$ is the probability density function of distances h chosen at random between the two line segments at a specified distance τ between each other. For the special case of $\tau=0$ — $f(h)$ is the density function of all possible distances within a line segment of length T which has the well known expression: $f(h) = \frac{2}{T} (1 - \frac{h}{T})$. The distribution function for a more general case of random distances between two line segments of equal length T shifted by the distance τ is derived using general results by Ghosh (1951):

$$\begin{aligned} f_1(h) &= \frac{1}{T_{CA}} \left(1 + \frac{h-\tau}{T_{CA}} \right); & (\tau - T_{CA}) \leq h \leq \tau \\ f_2(h) &= \frac{1}{T_{CA}} \left(1 - \frac{h-\tau}{T_{CA}} \right); & \tau < h \leq (\tau + T_{CA}) \end{aligned} \quad (8)$$

When integrating this distribution in accordance with Eq. (7) three cases have to be distinguished namely i) $\tau=0$; ii) $0 < \tau \leq T_{CA}$; and iii) $\tau > T_{CA}$:

i) $\tau=0$;

$$\begin{aligned}\text{Cov}_A(0) &= \text{Var}_A = \sigma_Z^2 \left\{ \int_{-T_{cA}}^0 \rho(-h) f_1(h) dh + \int_0^{T_{cA}} \rho(h) f_2(h) dh \right\} \\ &= 2\sigma_Z^2 \int_0^{T_{cA}} \rho(h) f_2(h) dh\end{aligned}\quad (9a)$$

ii) $0 < \tau \leq T_{cA}$;

$$\text{Cov}_A(\tau) = \sigma_Z^2 \left\{ \int_{-(T_{cA}-\tau)}^0 \rho(-h) f_1(h) dh + \int_0^{\tau} \rho(h) f_1(h) dh + \int_{\tau}^{T_{cA}+\tau} \rho(h) f_2(h) dh \right\}\quad (9b)$$

iii) $\tau > T_{cA}$;

$$\text{Cov}_A(\tau) = \sigma_Z^2 \left\{ \int_{\tau-T_{cA}}^{\tau} \rho(h) f_1(h) dh + \int_{\tau}^{\tau+T_{cA}} \rho(h) f_2(h) dh \right\}\quad (9c)$$

Let us exemplify the derivations above by assuming a simple exponential correlation function:

$$\rho(h) = \exp(-h/k)\quad (10)$$

where k is a time constant. The following expression is derived:

i) $\tau=0$;

$$\text{Cov}_A(0) = \gamma(A) = \sigma_A^2 = 2\sigma_Z^2 \left(\frac{k}{T_{cA}} \right)^2 \left\{ \frac{T_{cA}}{k} + e^{-T_{cA}/k} - 1 \right\}\quad (11a)$$

ii) $0 < \tau \leq T_{cA}$;

$$\text{Cov}_A(\tau) = 2\sigma_Z^2 \left(\frac{k}{T_{cA}} \right)^2 \left\{ \frac{T_{cA} - \tau}{k} + \frac{1}{2} \left(e^{(\tau - T_{cA})/k} + e^{-(\tau + T_{cA})/k} \right) - e^{-\tau/k} \right\} \quad (11b)$$

iii) $\tau > T_{cA}$;

$$\text{Cov}_A(\tau) = \sigma_Z^2 \left(\frac{k}{T_{cA}} \left\{ 1 - e^{-T_{cA}/k} \right\} \right)^2 e^{-(\tau - T_{cA})/k} \quad (11c)$$

5 The time constant k characterises the scale of persistence in the inflow to the river system i.e. the hillslope flow is approximated by a simple linear reservoir. This constant thus describes the dynamics of the runoff process in contrast to the constant K introduced in Eq. (1), which describes the characteristic spatial scale of variation in runoff formation. The two limiting cases of Eqs. (11a–c) are when $k \rightarrow 0$ and $k \rightarrow \infty$,
10 respectively. In the first case the correlation function Eq. (9) turns into a Dirac's delta function $\delta(\lambda)$ for $\lambda=0$, i.e. the characteristic of a process without memory. Applying the relationships Eqs. (12a–c) to this situation we find:

i) $\tau=0$;

$$\text{Cov}_A(0) = \sigma_A^2 = \sigma_Z^2 / T_{cA} \quad (12a)$$

15 ii) $0 < \tau \leq T_{cA}$;

$$\text{Cov}_A(\tau) = \sigma_Z^2 \frac{T_{cA} - \tau}{T_{cA}^2} \quad (12b)$$

iii) $\tau > T_{cA}$;

$$\text{Cov}_A(\tau) = 0 \quad (12c)$$

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i.e. a moving average process over a time period T_{cA} . For the second case with a very large memory (k large compared to T_{cA}) the variance is constant equal to σ_Z^2 .

Equation (11a), with Eq. (12a) as a special case, is thus the variance function for instantaneous runoff from an area of size A . It describes how the variance of runoff alters with this size (time of concentration T_{cA}). Taking the square root of this expression yields the corresponding reduction factor. In a system terminology it represents a combination between linear reservoirs entering into a linear channel or as first order autoregressive processes combined with a moving average one. Equations (11b, c) (Eqs. 12b, c) in their turn represent the auto-covariance functions of instantaneous runoff of a basin of size A .

Hydrological observations of runoff are, as a rule, limited to daily averages and the derived expressions are not directly applicable as they concern instantaneous runoff. For the observed data for a certain duration D we need to integrate these equations in the time domain to this specific duration. We will omit detailed derivations in the text as the principle is the same as described earlier, i.e. using Eq. (7) to develop frequency functions of distances between line segments (river branches, time intervals) $f(h)$ and then integration schemes in accordance with Eq. (8). The resulting equations of such derivations are shown in Appendix B for the variance functions for a basin of size A and duration D (B1), the auto-covariance function for a basin of size A and time lags nD , $n=0,1,\dots$ (B2), and cross-covariance function between two areas with sizes $A1$ and $A2$ and time lags nD , $n=0,1,\dots$ (B5). Some intermediate results are also shown. To simplify the notations dimensionless variables $\delta=D/k$ and $\eta=T_{cA}/k$ replace the duration D (time scale) and T_{cA} (space scale).

The random model, in principle, allows a reproduction of all statistical characteristics of the runoff data from the Moselle basin referred to in the first section of this paper. The theoretical model shows how they are interrelated. The parameters of the model can be determined by fitting the respective theoretical function to the empirical ones, established from the observed data. We will concentrate these efforts to the variance function (B1), which expresses the dependence in the variance of the time of concen-

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tration T_{cA} (area) and time duration D . The other functions will be used as a diagnostic test of model assumptions.

5 Fitting the model for the normalized variable

The second order moment of the normalized variable Z equals the coefficient of variation of the original variable X averaged. Figure 3 showed the dependence of the coefficient of variation on area A estimated from the observed runoff data on basin area for different durations D (an hour, a day, five days, a month and a year). In case of so-called simple scaling this parameter should be constant but the pattern of variation seen is indeed complex. In principle we should be able to model the behaviour in accordance with the variance function (B1), depending on the dimensionless variables $\delta = D/k$ and $\eta = T_{cA}/k$, respectively, and containing one parameter σ_Z equal to the coefficient of variation for instantaneous inflow to a river branch. Following the development along different branches a mirrored pattern of the variation of the first order moment (although with an increased scatter) can be identified, i.e. high runoff values show low coefficient of variation and vice versa. It was also found that the decay in the time dependence was poorly described by the assumed simple exponential function for the instantaneous point correlation function (Eq. 10). Two time scales could be identified (cf. Fig. 4) – one in the order of a day and another in the order of a month, with a domination of the first one for short time lags. To account for this fact the point exponential correlation function is modified as $\rho(h) = w \exp(-h/k_1) + (1-w) \exp(-h/k_2)$, where w is a weight coefficient. The variance function (B1) needs to be modified accordingly. As all operations for the derivation of this function from the point correlation function are linear the modified variance function is derived as two weighted components as in B1 with weights w and $(1-w)$ and with time variables $\delta_1 = D/k_1$, $\delta_2 = D/k_2$ and time of concentration variables $\eta_1 = T_{cA}/k_1$ and $\eta_2 = T_{cA}/k_2$, respectively.

For the variables $\eta_i, i=1,2$ and the point standard deviation σ_Z the following relations are proposed to take account of the dependence on basin area and mean runoff,

respectively:

$$\begin{aligned}\eta_1 k_1 &= \eta_2 k_2 = T_{cA} = a + b \ln(A) \\ \sigma_Z &= c(1 + d \times m_x)\end{aligned}\quad (13)$$

It is assumed that the time of concentration is proportional to the logarithm of the basin area A and that the coefficient of variation is linearly proportional to the mean annual runoff m_x .

We thus obtain a model for the description of the dependence of the coefficient of variation on basin area A and duration D containing seven parameters, namely w, k_1, k_2, a, b, c, d . Some of these parameters might vary with location, and should therefore develop along river branches. Here we for the time being assume a set of global parameters for the whole Moselle basin and determine them so that an optimal fit in the least square sense is obtained with the scatter of data in Fig. 6. A downhill simplex method (Press et al., 1992, p. 326–330) was applied for the search of optimal parameters, with the following result:

$$\begin{aligned}w &= 0.80 \\ \delta_1 &= D/0.63 \\ \delta_2 &= D/40.0 \\ \eta_1 k_1 &= \eta_2 k_2 = T_{cA} = -10.44 + 2.77 \ln(A); A > 43.5 \\ \sigma_Z &= 2.70(1 - 0.012 m_x)\end{aligned}\quad (14)$$

The estimated parameters confirm the impression of a decrease in the coefficient variation with the increase of the mean runoff and the existence of two characteristic time scales of the correlation function one a little less than a day and the other about one and a half month. The estimation error obtained for this set of parameters was 0.29.

The parameters allow now an estimation of the coefficient of variation as a function of basin area A and duration D . The resulting relationships are shown in Fig. 6 where observed values are compared with those estimated. The estimated is done for each grid cell in the digital map forming the grey background of the graphs. The proposed model is able to describe the main features of the dependence of the coefficient of

variation C_v of both the area and the duration. Some outlying observations of high C_v are, however, noted along branches with a low runoff contribution. The corresponding maps can be constructed and an illustration for C_v for $D=1$ day and $D=1$ month is found in the Appendix A.

6 Testing auto- and cross-correlation

The approach developed here is based on the existence of a covariance function along the river network. We have here chosen to determine the basic parameters in this covariance structure through the dependence of the coefficient of variation on the area A and duration D . It thus remains to be shown that the main features of this covariance structure are reproduced. Here we will do this by comparing the empirical statistical properties shown earlier in Figs. 4a to d. and those obtained by the model with the parameters determined in the previous section. The covariance includes duration as well as area, which would mean that we should be able to model the autocorrelation at a site in a river for different durations D , as well as the cross-correlation between sites along a river branch.

A comparison of empirical and model correlation functions carried out here should not be seen in the light of a formal test. The theoretical derivations developed herein are in its infancy and not yet ready for such formal procedures. This comparison is rather to be seen as a first diagnostic to indicate if the assumptions make sense. We will also here use branch 4 as an illustration when comparing the theoretical derivation with the empirical functions in Fig. 4. Figure 7a shows the estimated autocorrelation (B2) for durations of an hour, a day, five days and a month for the central site on this branch, and Fig. 7b the autocorrelation function for one day duration for all stations along the branch. Figure 7c in a similar way shows the estimated cross-correlation function (B5) between the central and the outlet sites for different durations and Fig. 7d the cross-correlation function between the outlet site and upstream stations along the same river branch for a duration of one day.

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In general the agreement is acceptable. The empirical and modelled correlation functions show the same main features. There are of course details both at very small and large time scales that are not yet grasped by the model. It describes well the behaviour of the empirical autocorrelation functions for different durations in time. However, the model is not able to reproduce the details of the spatial nonsystematic dependence of the empirical correlation functions along branches. The deviation between the model and empirical functions is largest in small basins. This can to a certain extent be traced back to the empirical relation of the time of concentration variable to $\ln(A)$ (Eq. 14), which contains a lower bound ($\sim 43 \text{ km}^2$).

We note that it was necessary to compose the theoretical model of two parts – one for small and the other for large time lags represented by the two parameters k_1 and k_2 , respectively and a weight coefficient w . For the time being we assumed these parameters to be the same for the whole of the Moselle basin. For larger sub-basins this can be an acceptable approximation. For small basins with specific local conditions, this might not be the case especially for the quickly decaying part of the correlation functions connected to the parameter k_1 . This is confirmed for some few small basins utilised here (a600101 and a605102 along branch 4). We interpret the large-scale component as a characteristic of the baseflow and seasonal climatic conditions for the region. The slow decaying part of the correlation functions related to the parameter k_2 is rather stable. The weight coefficient w balancing the local and regional influence seems also to be relatively constant for larger basins, while small ones may show deviations from this regional value possibly due to different baseflow contributions. The parameters a , b , c and d are all interpreted to be of a regional character for the time being.

7 Discussion and conclusions

The problem of mapping runoff characteristics herein has been divided into two steps. Firstly the first order moment (the long term mean value) is analysed and mapped ap-

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plying an interpolation procedure for river runoff proposed by Gottschalk (1993b). In a second step a simple random model for the river instantaneous point runoff process normalised with respect to the long term mean is proposed, which allows the derivation of the time-space variance-covariance function of the inflow to the river network. This function is then used to predict how the coefficient of variation develops from headwaters of the different river branches down to the river mouth. The runoff characteristics in a downstream point, here the first and second order moments, are thus derived by integration over the upstream contributing river network. The results are thus consistent in this respect and furthermore are able to reproduce the main features of the space-time covariance within a basin. The observation data are handled in the frame of a hydrological information system HydroDem (Leblois and Sauquet, 2000), which allows displaying the results either in the form of area dependence of moments along the river branches to the basin outlet or as a map of the variation of the moments across the basin space.

The runoff variation across space is first of all explained by the variation in the average runoff formation. For the Moselle River basin (French part), studied here, the characteristic length scale of this process has been identified as 50 km. The basic parameters in the stochastic model for the second order moment express the characteristic scales in time. Two characteristic temporal scales were identified – one related to the dynamics of the runoff formation process in the order of a day, k_1 , and another related to the persistence in baseflow and climatic condition in the order of a month, k_2 .

While the influence of the time scale on auto- and cross-correlation functions is well in agreement between empirical and modelled data, the influence of the spatial scale, expressed as a “time of concentration” is more complex and not yet fully grasped by the model. The “time of concentration” is here a measure of the variance reduction in space, which corresponds to a similar reduction in time. A possible reason for the poorer fit might be the assumption of one global set of parameters for the whole Moselle basin. Further studies are needed to verify if allowing the parameters vary

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between river branches (in particular the small time scale parameter k_1 and the weight coefficient w balancing the influence of the two time scales) can improve the results. However, aggregation rules need to be elaborated for this purpose. Another uncertainty is related to the assumption of the time-space correlation function Eq. (5) utilised here. The theoretical implications of this needs to be further elaborated.

A random model, like the one developed here, involves directly the statistical properties of runoff and their dependence on time and space. Traditionally, in the so-called derived distribution function approach (Eagleson, 1972), the ambition is to reduce the parameters in such a random model to those with a physical or at least a conceptual interpretation. So far the physical/conceptual models used for this purpose are quite simplistic. A simplistic approach is also adopted here where the basic conceptual model consists of a linear reservoir combined with a linear channel. The advantage is the ability of the model to preserve the statistical properties of the observed data in space and time. The simple representation might also improve the performance of the estimation method, a topic that for the time being has not been studied in any depth.

Distributed modelling is another expanding activity in hydrology that lately also has been used for mapping purposes (e.g. Beldring et al., 2002). A fair comparison between the two approaches is possible, however, only if the diagnostics used for evaluating the performance of distributed models is extended to the ability of preserving of time-space statistical and scale dependence. The preservation of time-space statistical and scale properties is of outmost importance for further developments for modelling and mapping of runoff for different durations as herein, and extremes (floods and drought) for different durations.

Appendix A

See Fig. A1.

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Appendix B

B1 Variance function for a basin area A and duration D .

$$\delta = D/k; \eta = T_{cA}/k; \tau' = \tau/k;$$

$$\begin{aligned} f_1(\lambda) &= \frac{1}{\delta} \left(1 + \frac{\lambda - \tau'}{\delta} \right); & (\tau' - \delta) \leq \lambda \leq \tau' \\ f_2(\lambda) &= \frac{1}{\delta} \left(1 - \frac{\lambda - \tau'}{\delta} \right); & \tau' < \lambda \leq (\tau' + \delta) \end{aligned} \quad (B1)$$

5 i) $0 < \delta \leq \eta$;

$$\gamma_A(\delta) = 4\sigma_Z^2 (\delta\eta)^{-2} \left\{ \frac{\delta^2\eta - \delta^3}{2} + \frac{1}{2} \left(e^{(\delta-\eta)} + e^{-(\delta+\eta)} \right) + 1 - \delta - e^{-\eta} - e^{-\delta} \right\} \quad (B2a)$$

ii) $\delta > \eta$;

$$\gamma_A(\delta) = 4\sigma_Z^2 (\delta\eta)^{-2} \left\{ \frac{\delta\eta^2 - \eta^3}{2} + \frac{1}{2} \left(e^{-(\delta-\eta)} + e^{-(\delta+\eta)} \right) + 1 - \eta - e^{-\eta} - e^{-\delta} \right\} \quad (B2b)$$

B2 Auto-covariance function for a basin area A and time lag nD .

$$10 \delta = D/k; \eta = T_{cA}/k;$$

$$\begin{aligned} f(\lambda) &= \frac{1}{\delta} \left(\frac{\lambda}{\delta} - n + 1 \right); & (n-1)\delta \leq \lambda \leq n\delta \\ f(\lambda) &= \frac{1}{\delta} \left(n + 1 - \frac{\lambda}{\delta} \right); & n\delta \leq \lambda \leq (n+1)\delta \end{aligned} \quad (B3)$$

i) $(n+1)\delta \leq \eta$

$$\text{Cov}_A(n\delta) = 2\sigma_Z^2 (\delta\eta)^{-2} \left\{ \delta^2 (\eta - n\delta) + \left(e^{\delta/2} - e^{-\delta/2} \right)^2 \left(\frac{1}{2} e^{-(n\delta+\eta)} + \frac{1}{2} e^{(n\delta-\eta)} - e^{n\delta} \right) \right\} \quad (B4a)$$

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ii) $n\delta \leq \eta < (n+1)\delta$

$$\begin{aligned} \text{Cov}_A(n\delta) = & 4\sigma_Z^2 (\delta\eta)^{-2} \left\{ e^{-n\delta} + \frac{1}{4} \left(e^{-((n-1)\delta+\eta)} + e^{((n-1)\delta-\eta)} + e^{-((n+1)\delta+\eta)} + e^{-((n+1)\delta-\eta)} \right) \right. \\ & - \frac{1}{2} \left(e^{-(n-1)\delta} + e^{-(n+1)\delta} + e^{(n\delta-\eta)} + e^{-(n\delta+\eta)} \right) - \eta/2 + \eta^3/12 \\ & \left. + \delta(1+n)/2 + \left(\delta^2\eta \left(n^2 + 2n - 1 \right) - \delta\eta^2(n+1) - \delta^3 \left(\frac{1}{3}n^3 + n^2 - n + \frac{1}{3} \right) \right) / 4 \right\} \end{aligned} \quad (\text{B4b})$$

iii) $(n-1)\delta \leq \eta < n\delta$

$$\begin{aligned} \text{Cov}_A(n\delta) = & 4\sigma_Z^2 (\delta\eta)^{-2} \left\{ e^{-n\delta} + \frac{1}{4} \left(e^{-((n-1)\delta+\eta)} + e^{((n-1)\delta-\eta)} + e^{-((n-1)\delta-\eta)} + e^{-((n+1)\delta+\eta)} \right) \right. \\ & - \frac{1}{2} \left(e^{-(n-1)\delta} + e^{-(n+1)\delta} + e^{(-n\delta+\eta)} + e^{-(n\delta+\eta)} \right) + \eta/2 - \eta^3/12 \\ & \left. + \delta(1-n)/2 - \left(\delta^2\eta \left(n^2 - 2n + 1 \right) - \delta\eta(n-1) - \delta^3 \left(\frac{1}{3}n^3 - n^2 + n - \frac{1}{3} \right) \right) / 4 \right\} \end{aligned} \quad (\text{B4c})$$

iv) $(n-1)\delta > \eta$

$$\text{Cov}_A(n\delta) = \sigma_Z^2 \left\{ (\delta\eta) (1 - e^{-\eta}) \left(e^{\delta/2} - e^{-\delta/2} \right) \right\}^2 e^{-(n\delta-\eta)} \quad (\text{B4d})$$

B3 Covariance between two nested catchments A_1 and A_2 ($A_1 < A_2$)

(Gottschalk, 1993a)

$$\eta_1 = T_{CA1}/k; \quad \eta_2 = T_{CA2}/k;$$

$$\begin{aligned} f_1(\lambda) &= \frac{1}{\eta_2} \left(1 + \frac{\lambda}{\eta_1} \right); & -\eta_1 \leq \lambda \leq 0 \\ f_2(\lambda) &= \frac{1}{\eta_2}; & 0 \leq \lambda \leq (\eta_2 - \eta_1) \\ f_3(\lambda) &= \frac{1}{\eta_2} \left(1 - \frac{\lambda}{\eta_1} \right); & (\eta_2 - \eta_1) < \lambda \leq \eta_2 \end{aligned} \quad (\text{B5})$$

$$\text{Cov}(A_1, A_2, 0) = 2\sigma_Z^2 (\eta_1\eta_2)^{-1} \left\{ \eta_1 + \frac{1}{2}e^{-\eta_1} + \frac{1}{2}(1-\eta_1+\eta_2) \left(e^{-\eta_2} - e^{-(\eta_2-\eta_1)} \right) - \frac{1}{2} \right\} \quad (\text{B6})$$

B4 Cross-covariance between two nested catchments A_1 and A_2 ($A_1 < A_2$) for instantaneous runoff with lag τ' .

$$\eta_1 = T_{cA1}/k; \eta_2 = T_{cA2}/k; \tau' = \tau/k;$$

$$\begin{aligned} f_1(\lambda) &= \frac{1}{\eta_2} \left(1 + \frac{\lambda - \tau'}{\eta_1} \right); & (\tau' - \eta_1) \leq \lambda \leq \tau' \\ f_2(\lambda) &= \frac{1}{\eta_2}; & \tau' \leq \lambda \leq (\eta_2 - \eta_1 + \tau') \\ f_3(\lambda) &= \frac{1}{\eta_2} \left(1 - \frac{\lambda - \tau'}{\eta_1} \right); & (\eta_2 - \eta_1 + \tau') < \lambda \leq (\eta_2 + \tau') \end{aligned} \quad (B7)$$

5 i) $\tau=0$;

$$\text{Cov}(A_1, A_2, 0) = 2\sigma_Z^2 (\eta_1 \eta_2)^{-1} \left\{ \eta_1 + \frac{1}{2} e^{-\eta_1} + \frac{1}{2} (1 - \eta_1 + \eta_2) \left(e^{-\eta_2} - e^{-(\eta_2 - \eta_1)} \right) - \frac{1}{2} \right\} \quad (B8a)$$

ii) $0 < \tau \leq \eta_1$

$$\text{Cov}(A_1, A_2, \tau) = 2\sigma_Z^2 (\eta_1 \eta_2)^{-1} \left\{ (\eta_1 - \tau) + \frac{1}{2} e^{-(\eta_1 - \tau)} + \frac{1}{2} (1 - \eta_1 + \eta_2) \left(e^{-(\eta_2 + \tau)} - e^{-(\eta_2 - \eta_1 + \tau)} \right) - \frac{1}{2} e^{-\tau} \right\} \quad (B8b)$$

iii) $\tau > \eta_1$;

$$10 \quad \text{Cov}(A_1, A_2, \tau) = \sigma_Z^2 (\eta_1 \eta_2) \left\{ e^{-(\eta_2 - \eta_1)} - e^{-\eta_2} \right\} \left\{ 1 - (1 - \eta_1 + \eta_2) e^{-\eta_2} \right\} e^{-(\tau - \eta_2)} \quad (B8c)$$

B5 Cross-covariance between two nested catchments A_1 and A_2 ($A_1 < A_2$) for runoff with time lag nD .

$$\delta = D/k; \eta_1 = T_{cA1}/k; \eta_2 = T_{cA2}/k;$$

$$\begin{aligned} f(\lambda) &= \frac{1}{\delta} \left(\frac{\lambda}{\delta} - n + 1 \right); & (n - 1)\delta \leq \lambda \leq n\delta \\ f(\lambda) &= \frac{1}{\delta} \left(n + 1 - \frac{\lambda}{\delta} \right); & n\delta \leq \lambda \leq (n + 1)\delta \end{aligned} \quad (B9)$$

i) $\delta \leq \eta_1, n=0$

$$\text{Cov}(A_1, A_2, 0) = 2\sigma_z^2 \left(\delta^2 \eta_1 \eta_2 \right)^{-1} \left\{ 1 + \delta^2 (\eta_1 - \delta/3) + e^{-(\eta_1 - \delta)} - e^{-\delta} - (\delta + 1) e^{-\eta_1 - \delta} \right. \\ \left. + (1 - \eta_1 + \eta_2) \left(e^{-(\delta + \eta_2)} - e^{-(\delta - \eta_1 + \eta_2)} \right) - (1 - \delta) (1 - \eta_1 + \eta_2) \left(e^{-\eta_2} - e^{-(\eta_2 - \eta_1)} \right) \right\} \quad (\text{B10a})$$

ii) $\delta > \eta_1, n=0$

$$\text{Cov}(A_1, A_2, 0) = 2\sigma_z^2 \left(\delta^2 \eta_1 \eta_2 \right)^{-1} \left\{ 1 + \eta_1^2 (\delta - \eta_1/3) + e^{(\eta_1 - \delta)} - e^{-\delta} - (\delta + 1) e^{-\eta_1 + \delta} - 2\eta_1 \right. \\ \left. + (1 - \eta_1 + \eta_2) \left(e^{-(\delta + \eta_2)} - e^{-(\delta + \eta_1)} \right) - (1 - \delta) (1 - \eta_1 + \eta_2) \left(e^{-\eta_2} - e^{-(\eta_2 - \eta_1)} \right) \right\} \quad (\text{B10b})$$

5 iii) $(n+1) \delta \leq \eta_1, n \geq 1$

$$\text{Cov}(A_1, A_2, nD) = 2\sigma_z^2 \left(\delta^2 \eta_1 \eta_2 \right)^{-1} \left\{ \delta^2 (\eta_1 - n\delta) + \frac{1}{2} \left(e^{\delta/2} - e^{-\delta/2} \right)^2 \left[e^{-(n\delta - \eta_2)} - e^{-n\delta} \right. \right. \\ \left. \left. + (1 - \eta_1 + \eta_2) \left(e^{-(n\delta + \eta_2)} - e^{-(\eta_2 - \eta_1 + n\delta)} \right) \right] \right\} \quad (\text{B10c})$$

iv) $(n-1)\delta > \eta_1, n > 1$

$$\text{Cov}(A_1, A_2, nD) = \sigma_z^2 \left(\delta^2 \eta_1 \eta_2 \right)^{-1} \left(e^{\delta/2} - e^{-\delta/2} \right)^2 \left(e^{-(\eta_2 - \eta_1)} - e^{-\eta_2} \right) \\ \left(1 - (1 - \eta_1 + \eta_2) e^{-\eta_2} \right) e^{-(n\delta - \eta_2)} \quad (\text{B10d})$$

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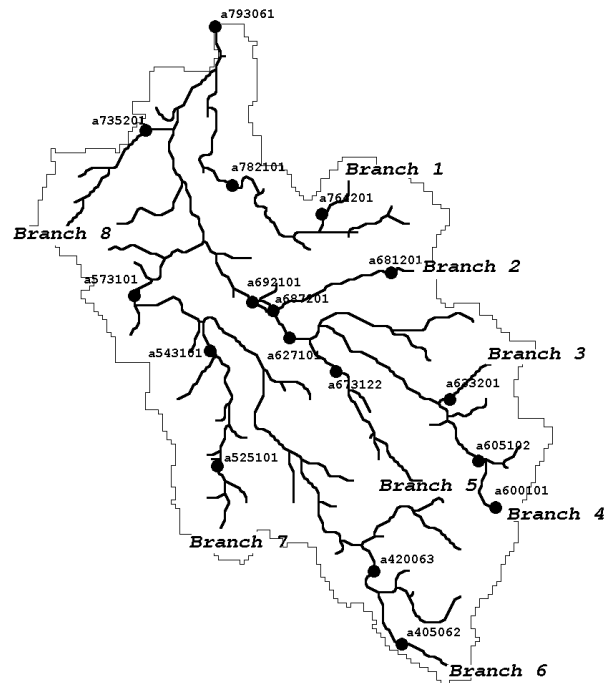


Fig. 1. Schematised river network of the Moselle basin down to Hauconcourt (a793061) with eight main branches and 17 gauging stations used in the study.

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L. Gottschalk et al.

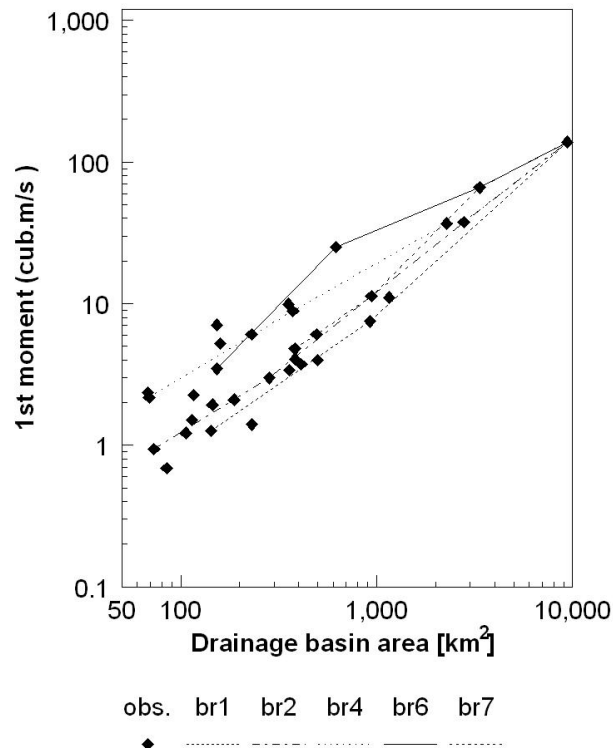


Fig. 2. The estimated first moments around zero of the Moselle discharge data plotted towards basin area A in a double logarithmic diagram. This first order moment (the long term mean values) does not depend on the duration D .

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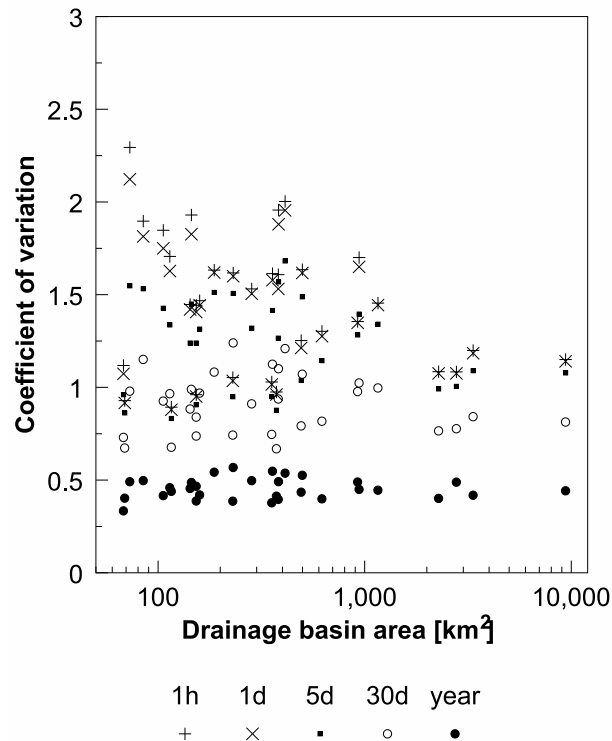


Fig. 3. The estimated coefficient of variation of runoff for the Moselle discharge data plotted towards basin area A for different durations D . This normalised second order moment decreases with the increasing duration from an hour (the highest values) to a day, five days, a month and a year (the lowest values).

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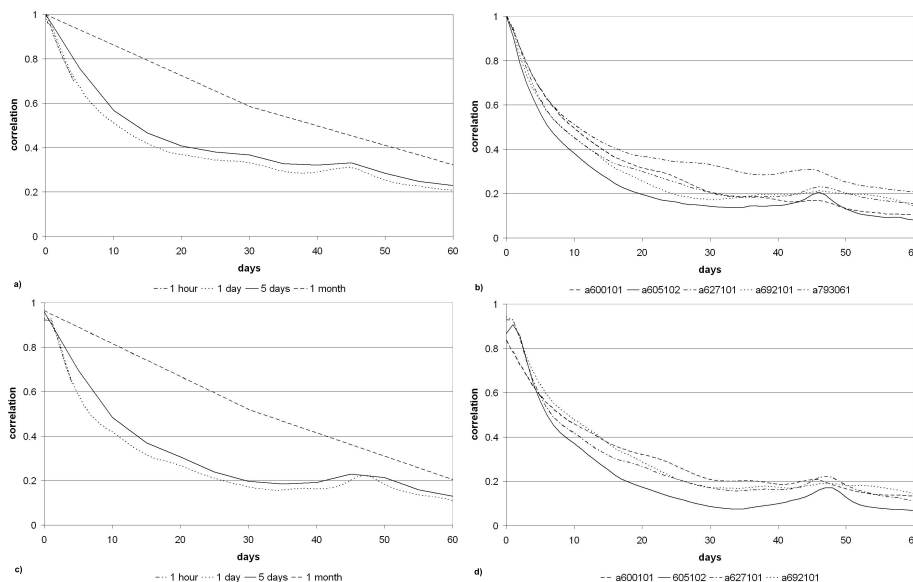


Fig. 4. Estimated empirical correlation functions in and along branch 4 (the Meurthe River) down to the outlet (station a793061): **(a)** the autocorrelation function for the central site (a627101) for different durations; **(b)** the same function for one hour duration for the five sites along branch 4; **(c)** the cross-correlation between the central and outlet sites for different durations and **(d)** the cross-correlation between the outlet and upstream sites. (In (a) and (c) the curves for 1 h and 1 day coincide except for very short time lags and the first one is therefore hidden behind the second one). There is no systematic difference between the headwater and downstream stations in graphs (b) and (c).

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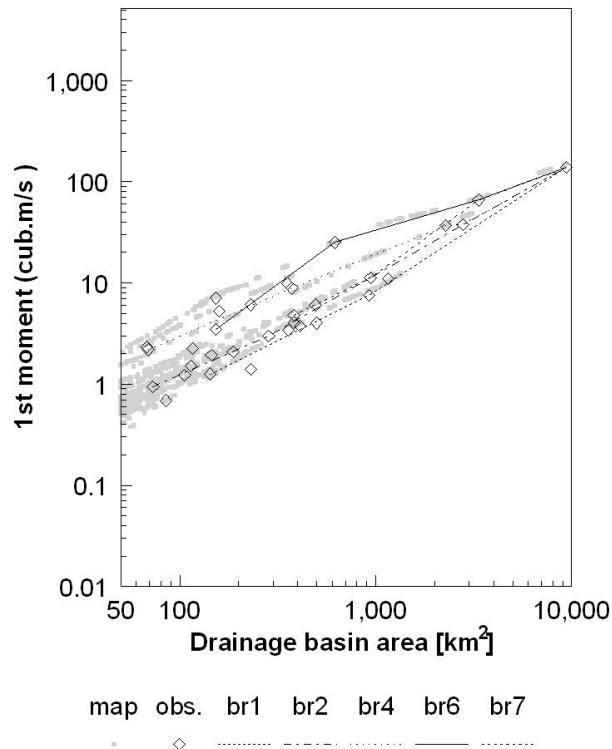


Fig. 5. The area dependence of the first order moment estimated and modelled at observations sites and mapped along the river net (cf. corresponding map in Appendix A). Area dependences develop along river branches towards a common value at the outlet.

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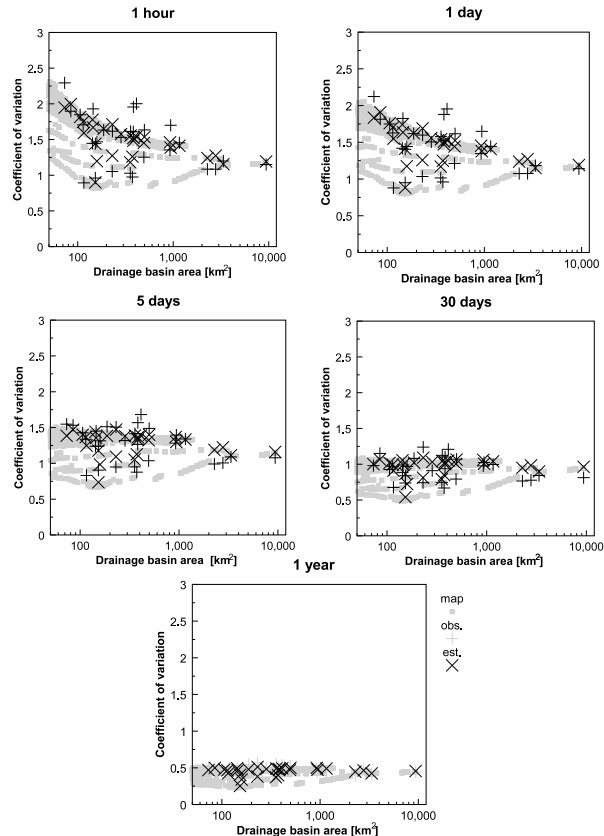


Fig. 6. Comparison between estimated and modelled area dependence in the coefficient of variation for different durations of the Moselle discharge data and the values for each grid cell in the digital map (cf. corresponding maps in Appendix A).

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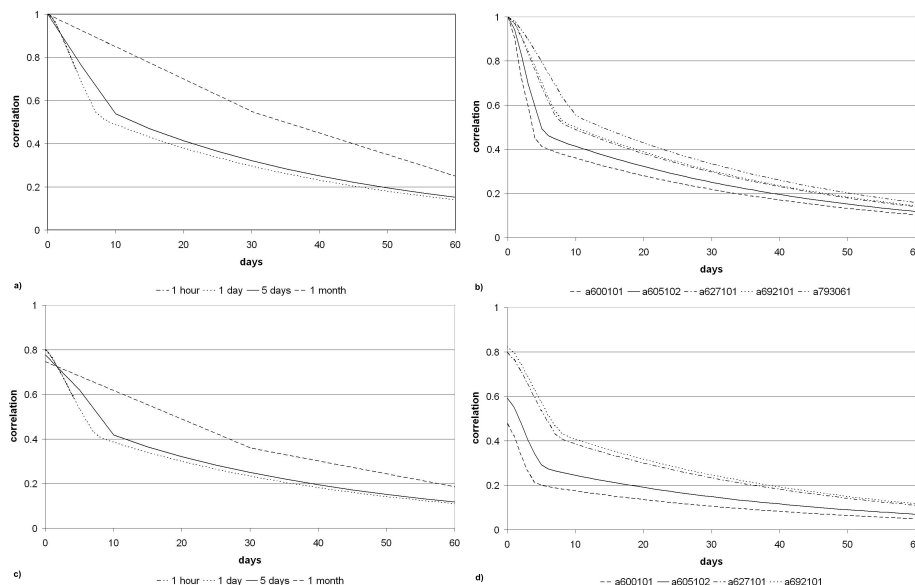


Fig. 7. Modelled correlation functions in and along branch 4 down to the outlet: **(a)** the autocorrelation function for the central site in this river branch for different durations; **(b)** the same function for one hour duration for the five sites along the same river branch; **(c)** the cross-correlation between the central and outlet sites for different durations and **(d)** the cross-correlation between the outlet and upstream sites. The corresponding empirical functions are illustrated in Fig. 4. (In (a) and (c) the curves for 1 h and 1 day coincide except for very short time lags and the first one is therefore hidden behind the second one). The mathematical model shows clear differences between headwater and downstream stations, which was not reflected in the empirical curves.

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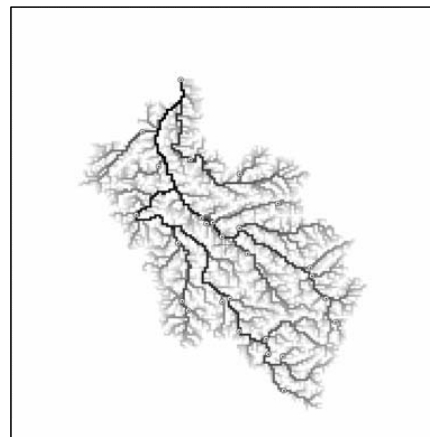
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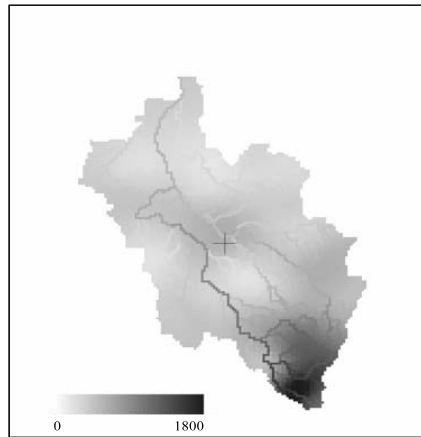
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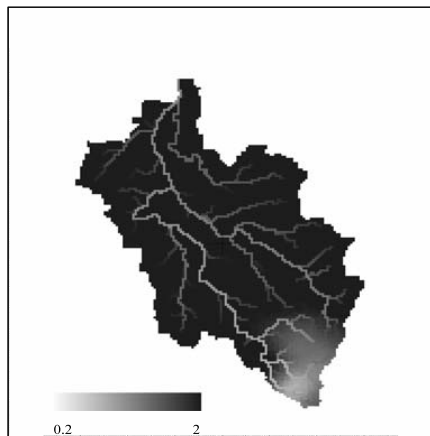
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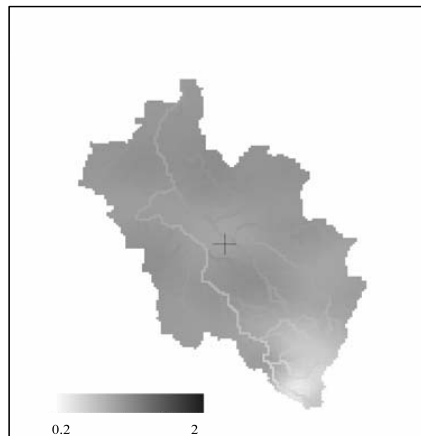
accumulated area



average runoff, mm/year



coefficient of variation for 1 day



coefficient of variation for 30 days

Fig. A1. Maps.

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